

Enhanced breaking of heavy quark spin symmetry

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Heavy quark spin symmetry is useful to make predictions on ratios of decay or production rates of systems involving heavy quarks. The breaking of spin symmetry is generally of the order of $\mathcal{O}(\Lambda_{\text{QCD}}/m_Q)$, with Λ_{QCD} the scale of QCD and m_Q the heavy quark mass. In this paper, we will show that a small S - and D -wave mixing in the wave function of the heavy quarkonium could induce a large breaking in the ratios of partial decay widths. As an example, we consider the decays of the $\Upsilon(10860)$ into the $\chi_{bJ}\omega$ ($J = 0, 1, 2$), which were recently measured by the Belle Collaboration. These decays exhibit a huge breaking of the spin symmetry relation were the $\Upsilon(10860)$ a pure $5S$ bottomonium state. We propose that this could be a consequence of a mixing of the S -wave and D -wave components in the $\Upsilon(10860)$. Prediction on the ratio $\Gamma(\Upsilon(10860) \rightarrow \chi_{b0}\omega)/\Gamma(\Upsilon(10860) \rightarrow \chi_{b2}\omega)$ is presented assuming that the decay of the D -wave component is dominated by the coupled-channel effects.

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A heavy quarkonium is a system consisting of a heavy quark and a heavy antiquark. The ground states and low-lying excited states below the open-flavor thresholds were well described in terms of potential quark models, e.g., the Godfrey–Isgur quark model [1], while the higher excited states are more complicated. The complexity comes from, e.g., the nearby strongly coupled thresholds, the existence of many new quarkonium-like states discovered in the last decade and so on. Because the heavy quark mass m_Q is much larger than the scale of quantum chromodynamics (QCD), Λ_{QCD} , the amplitude of changing the spin orientation of a heavy quark by interacting with soft gluons is small, suppressed by $\mathcal{O}(\Lambda_{\text{QCD}}/m_Q)$ relative to the spin-conserving case [2]. The resulting heavy quark spin symmetry (HQSS) [3] can lead to important observable consequences. On the one hand, heavy quarkonium states are organized into spin multiplets; on the other hand, the decay or production rate involving one heavy quarkonium can often be related to the one of its spin partners in the leading approximation. Breaking of HQSS is typically of the order of $\mathcal{O}(\Lambda_{\text{QCD}}/m_Q)$ or even higher. In this paper, we will argue that the HQSS breaking could be much larger in certain processes. To be specific, we will show that a small mixing of S - and D -wave heavy quarkonia could result in a significant breaking of the spin symmetry relations when the decay amplitude of the D -wave component is enhanced. As an example, we will calculate the processes $\Upsilon(10860) \rightarrow \chi_{bJ}\omega$ ($J = 0, 1, 2$). Measurements for these transitions were done by the Belle Collaboration very recently, and the results for the branching fractions are [4]

$$\begin{aligned}\mathcal{B}(\Upsilon(10860) \rightarrow \chi_{b0}\omega) &< 3.9 \times 10^{-3}, \\ \mathcal{B}(\Upsilon(10860) \rightarrow \chi_{b1}\omega) &= (1.57 \pm 0.22_{\text{stat.}} \pm 0.21_{\text{sys.}}) \times 10^{-3}, \\ \mathcal{B}(\Upsilon(10860) \rightarrow \chi_{b2}\omega) &= (0.60 \pm 0.23_{\text{stat.}} \pm 0.15_{\text{sys.}}) \times 10^{-3}.\end{aligned}\tag{1}$$

One sees that the branching fraction for the $\chi_{b1}\omega$ mode is larger than that for the $\chi_{b2}\omega$. Comparing the HQSS prediction on the ratio $\mathcal{B}(\Upsilon(5S) \rightarrow \chi_{b1}\omega)/\mathcal{B}(\Upsilon(5S) \rightarrow \chi_{b2}\omega) = 0.63$ assuming the $\Upsilon(10860)$ to be the $5S$ bottomonium state, see Eq. (6) below, with the observed value 2.62 ± 1.30 , the breaking is more than 100%. This is a very large spin symmetry breaking. As we will show later, a small mixture of a D -wave $\bar{b}b$ component in the $\Upsilon(10860)$ is able to cause the ratios of $\Gamma(\Upsilon(10860) \rightarrow \chi_{bJ}\omega)$ to be very different from the spin symmetry relations as observed.

Consequences of HQSS can be easily analyzed using heavy meson effective field theory (for a review, see Ref. [5]). Let us take the transitions from a vector heavy quarkonium into the $\chi_J\omega$ as an example, where χ_J is a P -wave heavy quarkonium with quantum numbers $J^{PC} = J^{++}$. Here we will use the two-component notation in Ref. [6] which is convenient for nonrelativistic processes with negligible recoil effect. The fields for the S -wave, P -wave and D -wave heavy quarkonium states are

denoted by J , χ^i and J^{ij} , respectively, which are $J = \vec{\psi} \cdot \vec{\sigma}$, $\chi^i = \sigma^j \left(\delta^{ij} \chi_0 / \sqrt{3} - \epsilon^{ijk} \chi_1^k / \sqrt{2} - \chi_2^{ij} \right)$, $J^{ij} = \frac{3}{2\sqrt{15}} \left(\psi_D^i \sigma^j + \psi_D^j \sigma^i \right) - \frac{1}{\sqrt{15}} \delta^{ij} \vec{\psi}_D \cdot \vec{\sigma}$ [5, 7–9], where $\vec{\sigma}$ are the Pauli matrices, and ψ , χ_J and ψ_D annihilate the S -, P - and D -wave heavy quarkonia, respectively. The states included in the above expressions have other spin partners which can be included as well, however, only the fields relevant for our discussion are shown.

Since the heavy quarkonia can be treated nonrelativistically, an expansion over low momenta can be done. To leading order of such an expansion, the Lagrangian for the decays of an S -wave or a D -wave heavy quarkonium into $\chi_J \omega$ reads

$$\mathcal{L}_{\chi\omega} = \frac{c_S}{2} \langle \chi^{i\dagger} J \rangle \omega^i + \frac{c_D}{4} \left(\langle \chi^{i\dagger} J^{ij} \rangle \omega^j + \langle \chi^{j\dagger} J^{ij} \rangle \omega^i \right), \quad (2)$$

where $\langle \rangle$ denotes the trace over the spinor space. With this Lagrangian, one is ready to obtain the ratios of decay widths of an excited S -wave heavy quarkonium into the $\chi_J \omega$ when the difference in phase space is neglected

$$\Gamma(\psi \rightarrow \chi_0 \omega) : \Gamma(\psi \rightarrow \chi_1 \omega) : \Gamma(\psi \rightarrow \chi_2 \omega) = 1 : 3 : 5. \quad (3)$$

The ratios are completely different if the initial state is a D -wave heavy quarkonium. In this case, one obtains

$$\Gamma(\psi_D \rightarrow \chi_0 \omega) : \Gamma(\psi_D \rightarrow \chi_1 \omega) : \Gamma(\psi_D \rightarrow \chi_2 \omega) = 20 : 15 : 1, \quad (4)$$

Therefore, the ratios of the decay widths of an excited heavy quarkonium into the $\chi_J \omega$ can be used to probe the spin structure of the initial state.

Replacing the ω by a photon, the above analysis still applies if we change the widths on the left side of Eqs. (3) and (4) by Γ/E_γ^3 with E_γ the photon energy in the rest frame of the initial state. The factor of the photon energy is required by gauge symmetry. As was shown long time ago in Ref. [10], the spin symmetry relations for the radiative transitions are generally in a quite good agreement with the experimental data, and the breaking of the spin symmetry relations is at the order of $\mathcal{O}(\Lambda_{\text{QCD}}/m_Q)$.

However, HQSS breaking for near-threshold vector quarkonium states could be enhanced due to the coupling to heavy meson pairs in a P -wave [11]. In the following, we will explore a different mechanism, and show that a small S - D mixing ¹ could result in a significant spin symmetry

¹ In our case of the decays $\Upsilon(10860) \rightarrow \chi_{bJ} \omega$, as will be shown later a mixing angle of $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_b^2) \sim 1^\circ$ is not sufficient. However, if the mixing angle can be enhanced to around 5° , which is still small, or larger, the huge HQSS breaking observed by the Belle Collaboration can be explained by the mechanism proposed here. Phenomenologically, the mixing angle for the $\Upsilon(10860)$ could be larger than 20° [12].

breaking if the decays of the D -wave component are enhanced by, for instance, coupled-channel effect as will be considered in the following.

Let us take the decays of the $\Upsilon(10860)$ into the $\chi_{bJ}\omega$ as a specific example. The $\Upsilon(10860)$ is often considered as the $5S$ vector bottomonium. It was argued that the HQSS breaking in the $\Upsilon(10860)$ decays into open-bottom mesons could be as large as 10% to 20% [13] (see also discussions in Ref. [14]). It is thus reasonable to assume that the wave function of the $\Upsilon(10860)$ contains a small mixture of a D -wave component, Υ_D . The decay amplitude can be written as

$$\mathcal{A}(\Upsilon(10860) \rightarrow \chi_{bJ}\omega) = \cos\theta \mathcal{A}_S + \sin\theta \mathcal{A}_D, \quad (5)$$

where θ is the mixing angle, and \mathcal{A}_S and \mathcal{A}_D are the decay amplitudes from the S -wave and D -wave components, respectively. One sees from Eqs. (3) and (4) that the ratios of the partial widths of the S -wave and D -wave components are distinct. When the phase space is taken into account, the corresponding ratios for the $\Upsilon(10860)$ decays in question are

$$\Gamma_0^S : \Gamma_1^S : \Gamma_2^S = 1 : 2.8 : 4.4, \quad (6)$$

and

$$\Gamma_0^D : \Gamma_1^D : \Gamma_2^D = 22.9 : 15.8 : 1 \quad (7)$$

respectively, where Γ_J represents $\Gamma(\Upsilon(10860) \rightarrow \chi_{bJ}\omega)$, and the index $S(D)$ means that only the $S(D)$ -wave component is considered.

Thus, if there is a mechanism to enhance the decay amplitude of the D -wave component relative to one of the S -wave component, a relatively small D -wave admixture can induce a sizable breaking of HQSS. In the following, we will assume that the decay width from the S -wave component is very small, and investigate the possibility of enhancing HQSS breaking due to such a mixing.

As analyzed in details in Ref. [8] for the transitions between two charmonium states with the emission of a pion or η -meson, some decay processes could be dominated by coupled-channel effects due to the coupling to the intermediate virtual heavy and anti-heavy mesons. Especially, the coupled-channel effect is the most important when both the vertices involving heavy quarkonia are in an S -wave. The mass of the $\Upsilon(10860)$ is only about 120 MeV below the threshold of the $B_1(5721)\bar{B}$. Thus, the decays of the D -wave component of the $\Upsilon(10860)$ could be dominated by meson loops as shown in Fig. 1. This is analogous to the radiative decays of the D -wave charmonia into the $X(3872)$ [15]. The hypothesis is based on a nonrelativistic power counting in terms of the velocity of the intermediate heavy mesons, denoted by v . Because both the initial and final

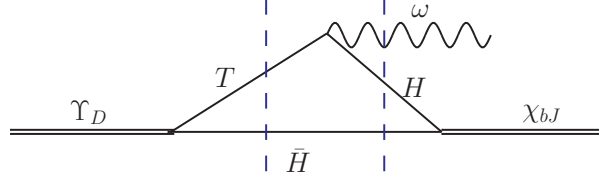


FIG. 1: The dominant decay mechanism for the D -wave component of the $\Upsilon(10860)$ into the $\chi_{bJ}\omega$. Here, Υ_D denotes the D -wave component, and T and $H(\bar{H})$ represent the bottom mesons with $s_\ell^P = \frac{3}{2}^+$ and $\frac{1}{2}^-$, respectively. The charge conjugated diagram is not shown but taken into account in the calculations. The vertical dashed lines indicate the two cuts operative in the process.

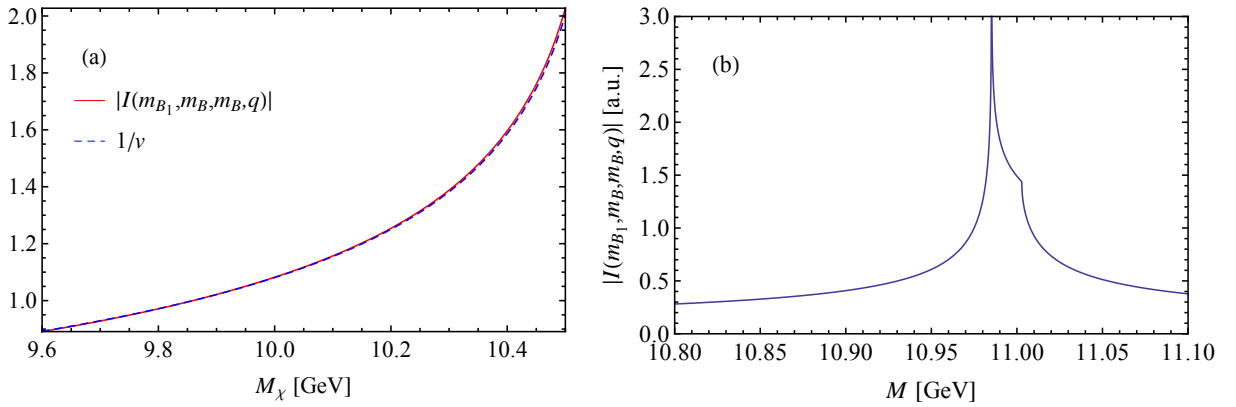


FIG. 2: (a) Illustration of the v^{-1} scaling of the scalar three-point loop function. The solid curve represents $|I(m_{B_1}, m_B, m_B, q)|$, see Eq. (A.2), with $M = M_{\Upsilon(10860)}$, and the dashed curve gives the inverse of the averaged velocity defined as $(v_1 + v_2)/2$ with $v_1 = \sqrt{2\mu_1(m_{B_1} + m_B - M_{\Upsilon(10860)})}/\bar{m}_1$, where μ_1 and \bar{m}_1 are the reduced and averaged masses of the B_1 and B , respectively, and $v_2 = \sqrt{(2m_B - M_\chi)/m_B}$. For comparison, both the loop function and $1/v$ are normalized at $M_\chi = M_{\chi_{b0}}$. (b) Dependence of $|I(m_{B_1}, m_B, m_B, q)|$ evaluated at $M_\chi = 2M_B$ on the mass of the initial state M .

heavy quarkonia are not far from the thresholds of the coupled heavy mesons, the intermediate heavy mesons are nonrelativistic with a velocity $v \ll 1$. For the diagram shown in Fig. 1, all three vertices are S -wave, and thus the loop amplitude is of the order $\mathcal{O}(v^5/(v^2)^3) = \mathcal{O}(v^{-1})$, where v^5 and $(v^2)^3$ account for the measure of the loop integral and three nonrelativistic propagators, respectively. Since the both the initial and final bottomonia are not far away from the threshold of the bottom meson pair, two unitary cuts are operative in this diagram, shown by the dashed vertical lines in Fig. 1. Each cut corresponds to a momentum, and therefore a velocity. As discussed in Appendix A, the velocity in the power counting corresponds to the average of the two velocities. This can be seen from a comparison of the scalar three-point loop function and the inverse of the averaged velocity as shown in Fig. 2(a). Notice that although the loop function scales as v^{-1} ,

it does not diverge even when both masses of the initial and final heavy quarkonium states are located at the corresponding thresholds. In Fig. 2(b), we show $|I(m_{B_1}, m_B, m_B, q)|$ evaluated at $M_\chi = 2M_B$ as a function of M . One sees that at threshold $M = m_{B_1} + m_B$, there is a cusp which is due to square-root singularity at the threshold; the sharp peak below the cusp is due to the Landau singularity discussed in Appendix A. For the processes in question, we have the averaged velocity $v \approx 0.26$. Therefore, the negative power of the small velocity provides an enhancement to the coupled-channel amplitudes. Thus, it is reasonable to assume that the decays of the D -wave component into the $\chi_{bJ}\omega$ are dominated by the loop diagrams as shown in Fig. 1, and the partial widths are not small. For more discussion of the power counting, we refer to Refs. [8, 16–19]. Next, we will perform an explicit calculation of the coupled-channel effect based on the mechanism shown in Fig. 1.

In the two-component notation, the fields for the S -wave ($s_\ell^P = \frac{1}{2}^-$) and P -wave ($s_\ell = \frac{3}{2}^+$) heavy mesons read $H_a = \vec{V}_a \cdot \vec{\sigma} + P_a$, and $T_a^i = P_{2a}^{ij} \sigma^j + \sqrt{2/3} P_{1a}^i + i\sqrt{1/6} \epsilon_{ijk} P_{1a}^j \sigma^k$, where P_a and V_a annihilate the pseudoscalar and vector heavy mesons, respectively, with $a = u, d$ labeling the light flavors, and P_{1a} and P_{2a} annihilate the axial and tensor heavy mesons, respectively. The fields annihilating their anti-particles are $\bar{H}_a = -\vec{V}_a \cdot \vec{\sigma} + \bar{P}_a$, $\bar{T}_a^i = -\bar{P}_{2a}^{ij} \sigma^j + \sqrt{2/3} \bar{P}_{1a}^i - i\sqrt{1/6} \epsilon_{ijk} \bar{P}_{1a}^j \sigma^k$. The properties of these fields under symmetry transformations can be found in Refs. [7, 15].

The Lagrangian, which is invariant under transformations of parity, charge conjugation, HQSS and Galilean invariance, for the coupling of the P -wave and D -wave heavy quarkonia to the $s_\ell = \frac{1}{2}^-$ and $s_\ell = \frac{3}{2}^+$ heavy mesons to leading order of the nonrelativistic expansion can be written as [7, 15, 20]

$$\mathcal{L}_{PD} = \frac{g_4}{2} \left\langle \left(\bar{T}_a^{j\dagger} \sigma^i H_a^\dagger - \bar{H}_a^\dagger \sigma^i T_a^{j\dagger} \right) J^{ij} \right\rangle + \frac{g_1}{2} \left\langle \chi^{i\dagger} H_a \sigma^i \bar{H}_a \right\rangle + \text{H.c.} \quad (8)$$

The S -wave coupling of the ω -meson to the S -wave and P -wave heavy mesons can be described by

$$\mathcal{L}_\omega = \frac{c_\omega}{2} \left\langle H_a^\dagger T_a^i - \bar{H}_a^\dagger \bar{T}_a^i \right\rangle \omega^i + \text{H.c.}, \quad (9)$$

where isospin symmetry is assumed.

Denoting the diagram shown in Fig. 1 by $[T\bar{H}H]$, the loops contributing to the processes $\Upsilon_D \rightarrow \chi_{bJ}\omega$ are listed in Tab. I. Using the Lagrangians given in Eqs. (8) and (9), the decay amplitudes can be easily obtained, and the explicit expressions are given in Appendix A. It is interesting to notice that if we take the same mass for the heavy mesons in the same spin multiplet, the spin symmetry relations are kept even if coupled channels are considered, that is, one would get the same ratios 20 : 15 : 1 for $\left| \mathcal{A}_{\Upsilon_D \rightarrow \chi_{bJ}\omega}^{\text{loop}} \right|^2$ as the ones in Eq. (4). This can be understood

TABLE I: Heavy meson loops contributing to the decays of the vector D -wave bottomonium into the $\chi_{bJ}\omega$. Here the charge conjugated ones are not listed but considered in the calculation.

Processes	$\Upsilon_D \rightarrow \chi_{b0}\omega$	$\Upsilon_D \rightarrow \chi_{b1}\omega$	$\Upsilon_D \rightarrow \chi_{b2}\omega$
Loops	$[B_1\bar{B}B], [B_1\bar{B}^*B^*], [B_2\bar{B}^*B^*]$	$[B_1\bar{B}B^*], [B_1\bar{B}^*B]$	$[B_1\bar{B}^*B^*], [B_2\bar{B}^*B^*]$

because the Lagrangians respect spin symmetry, and if we use degenerate masses, there will be no source for symmetry breaking.² When the physical masses for all the mesons are used, and the phase space difference is taken into account, the loop amplitudes will result in ratios slightly different from Eq. (7)

$$\Gamma_0^{\text{loop}} : \Gamma_1^{\text{loop}} : \Gamma_2^{\text{loop}} = 24.4 : 16.7 : 1. \quad (10)$$

One sees that the decays into the $\chi_{b0}\omega$ and $\chi_{b1}\omega$ are more enhanced than that into the $\chi_{b2}\omega$. The reason is that the $\Upsilon(10860)$ mass is closer to the $B_1\bar{B}$ threshold than to the $B_1\bar{B}^*$ one, cf. Table I. If we put the initial state at the mass of the $\Upsilon(11020)$, the heaviest known bottomonium, the ratios will be even larger, $27.5 : 18.4 : 1$.

With the above preparation, we can now show quantitatively how a significant HQSS breaking effect can be obtained from a small S - D mixing. In the following, we will assume that the decays of the D -wave component into the $\chi_{bJ}\omega$ are saturated by the triangle diagrams as discussed above. Because the S - D mixing is due to the tensor force between the heavy quark and antiquark, it is of $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_b^2)$, which corresponds to the mixing angle $\lesssim 1^\circ$ if Λ_{QCD} is taken to be of the order of a few hundreds MeV. However, as pointed out in Ref. [12], for highly excited bottomonia, the mass difference between the $(n+1)S$ and the nD states is small so that the mixing could be much larger. The phenomenological value for the $\Upsilon(4S)$ - $\Upsilon(3D)$ mixing angle extracted from the dielectron width is as large as $27^\circ \pm 4^\circ$, and the $5S$ - $4D$ mixing angle is of a similar size [12]. Indeed, if we take $\theta = 1^\circ$ and adjust the strength of the decay amplitudes of the S -wave and D -wave components to get the central values of $\Gamma(\Upsilon(10860) \rightarrow \chi_{b1}\omega) = (86 \pm 47) \text{ keV}$ and $\Gamma(\Upsilon(10860) \rightarrow \chi_{b2}\omega) = (33 \pm 23) \text{ keV}$, one would get an unreasonably large width for the D -wave component: two solutions are obtained

² This provides a simple method to calculate the HQSS relations for partial decay widths of processes involving hadronic molecules of a pair of heavy mesons, and the results in, e.g., Ref. [21] calculated using 6- j and 9- j symbols can be checked in this way. Without taking into account the phase space factors which include E_γ^3 for the radiative decays and setting mesons in the same spin multiplet to be degenerate, the ratios for the decay widths, Γ_J^{hm} , of a 1^{--} bottomonium-like hadronic molecule into $\chi_{bJ}\omega/\gamma$ are as follows: $\Gamma_0^{\text{hm}} : \Gamma_1^{\text{hm}} : \Gamma_2^{\text{hm}} = 3 : 1 : 0$ (for $B_1\bar{B}$), $1 : 12 : 5$ (for $B_1\bar{B}^*$), and $5 : 0 : 1$ (for $B_2\bar{B}^*$).

for $\Gamma_{D0} \equiv \Gamma(\Upsilon_D \rightarrow \chi_{b0}\omega) = 604 \text{ MeV}$ or 75 MeV . These two values correspond to the ratio of the decay amplitude of the S -wave component over that of the D -wave component $|\mathcal{A}_S/\mathcal{A}_D| = 0.002$ and 0.011 , respectively. These widths seem too large for an OZI-suppressed transition. Increasing the angle to 5° , they become much more reasonable— $\Gamma_{D0} = 24 \text{ MeV}$ or 3 MeV corresponding to $|\mathcal{A}_S/\mathcal{A}_D| = 0.008$ and 0.055 , respectively. For a mixing angle as large as 20° , one gets $\Gamma_{D0} = 1.6 \text{ MeV}$ or 0.2 MeV corresponding to $|\mathcal{A}_S/\mathcal{A}_D| = 0.034$ and 0.23 , respectively. In this regard, our explanation of the large HQSS breaking in the partial decay widths in Eq. (1) requires the mixing angle between the $5S$ and the $4D$ states to be at least around 5° . One should also notice that according to the power counting of nonrelativistic QCD, the mixing is of the order $v_b^2 \approx 0.1$ [22], where v_b is the velocity of the bottom quark in bottomonium. In this sense, a mixing angle of $\mathcal{O}(10^\circ)$ is natural. To be specific, let us take the mixing angle $\theta = 5^\circ$ for instance, which corresponds to $\sin \theta = 0.087$ and an S -wave dominance in the wave function. In Fig. 3 (a), we show the dependence of the ratios defined as

$$R_{02} = \frac{\Gamma(\Upsilon(10860) \rightarrow \chi_{b0}\omega)}{\Gamma(\Upsilon(10860) \rightarrow \chi_{b2}\omega)}, \quad R_{12} = \frac{\Gamma(\Upsilon(10860) \rightarrow \chi_{b1}\omega)}{\Gamma(\Upsilon(10860) \rightarrow \chi_{b2}\omega)} \quad (11)$$

on $|\mathcal{A}_S/\mathcal{A}_D|$ for $\theta = 5^\circ$. Because the interference between the S -wave and D -wave components can be either constructive or destructive, there are two possible solutions for each ratio. It is obvious that the variation is dramatic at small values of $|\mathcal{A}_S/\mathcal{A}_D|$ due to interference. This is because the contribution of the D -wave component is suppressed by the small mixing angle, and the S - D interference controls the results. Increasing $|\mathcal{A}_S|$, the contribution from the D -wave component diminishes, and the ratios approach those given in Eq. (6). We thus expect that for small values of $|\mathcal{A}_S/\mathcal{A}_D|$ the ratios would be very different from spin symmetry ones for the $\Upsilon(5S)$ given in Eq. (6). Figure 3 (b) shows the dependence on $\cos \theta$ for fixed $|\mathcal{A}_S/\mathcal{A}_D| = 0.05$.

We want to emphasize that the mixing angle and $\mathcal{A}_S/\mathcal{A}_D$ always appear together, and thus cannot be fixed from the measured branching fractions. However, when one of the ratios R_{02} or R_{12} is measured, the other can be predicted as shown in Fig. 4, and the uncertainty should be of $\mathcal{O}(v)$. In the figure, the Belle results $R_{12} = 2.62 \pm 1.30$ and $R_{02} < 13.3$ obtained from Eq. (1) are shown as the shaded area. Fixing R_{12} to the measured range, we predict two possible ranges for R_{02} ,

$$R_{02} = 7.1 \pm 2.1 \pm 1.8, \quad \text{or} \quad R_{02} = 0.19 \pm 0.17 \pm 0.05, \quad (12)$$

where the first uncertainty is propagated from the measured uncertainty of R_{12} , and the second one from $v = 0.26$ is inherent in our nonrelativistic framework. Both ranges are consistent with

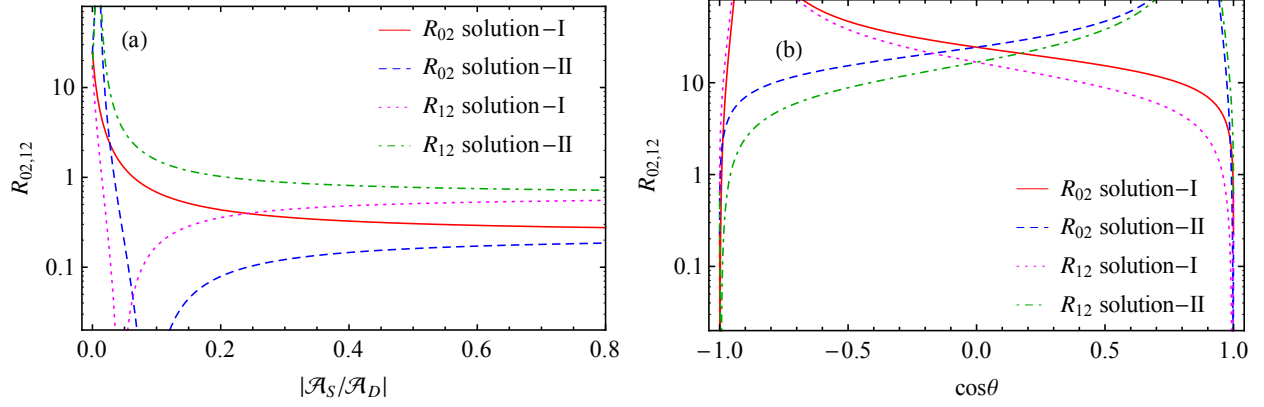


FIG. 3: Dependence of the ratios R_{02} and R_{12} defined in Eq. (11) on $|\mathcal{A}_S/\mathcal{A}_D|$ for $\theta = 5^\circ$ (a), and on $\cos\theta$ for $|\mathcal{A}_S/\mathcal{A}_D| = 0.05$ (b).

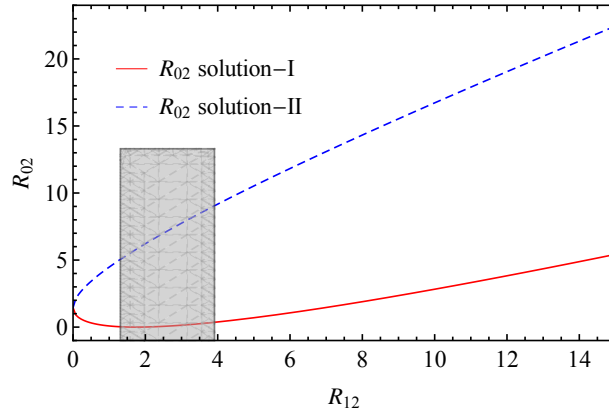


FIG. 4: Prediction of R_{02} for a given value of R_{12} . The shaded area corresponds to the range reported by the Belle Collaboration [4].

the Belle upper limit, and an examination of the HQSS breaking mechanism proposed here urges an improved measurement, especially for the R_{02} . This can be done at the future super- B factory. Similarly, we can make predictions for the decays $\Upsilon(11020) \rightarrow \chi_{bJ}\omega$. The curves are similar with slightly larger values.

To summarize, we have discussed a new mechanism to produce a sizable breaking of HQSS. We showed that a small S - D mixing for the vector heavy quarkonium could result in a much larger spin symmetry breaking effect. In order for this mechanism to work, the decays of the D -wave component should be enhanced in comparison with that of the S -wave one. As an example, we studied the decays $\Upsilon(10860) \rightarrow \chi_{bJ}\omega$ in details. The decays of the D -wave component of the $\Upsilon(10860)$ are assumed to be dominated by the coupled-channel effects due to S -wave coupling to nearby thresholds of a P -wave and an S -wave heavy meson pair. It was found that a mixing

angle of $\mathcal{O}(1^\circ)$ would result in a too large width for the D -wave component, and $\theta \gtrsim 5^\circ$, i.e. $\sin\theta \gtrsim 0.087$, is needed to explain the observed widths of the decays into the $\chi_{b1}\omega$ and $\chi_{b2}\omega$. It is noticeable that a $\mathcal{O}(10\%)$ D -wave component, though needs an additional explanation for bottomonium states [12], is sufficient to explain an HQSS breaking $\gtrsim 100\%$ in the ratios of the partial decay widths. In particular, when one of the ratios of branching fractions for the processes $\Upsilon(10860, 11020) \rightarrow \chi_{bJ}\omega$ is measured, the other can be predicted independent of the mixing angle. Using the Belle measurement for R_{12} , two possible ranges of R_{02} were predicted. The prediction can be examined at the future super- B factory. Such measurements will be important to better understand the spin symmetry breaking as well as the nature of the $\Upsilon(10860)$ and $\Upsilon(11020)$.

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Appendix A: Decay amplitudes and Landau singularities of the three-point loop function

The explicit expressions for the decay amplitudes for the D -wave component through the $[T\bar{H}H]$ triangle diagrams are given by

$$\begin{aligned} \mathcal{A}_{\Upsilon_D \rightarrow \chi_{b0}\omega}^{\text{loop}} &= -N \frac{2\sqrt{5}}{3} g_1 g_4 c_\omega \vec{\epsilon}_{\Upsilon_D} \cdot \vec{\epsilon}_\omega [6I(m_{B_1}, m_B, m_B, q) + I(m_{B_1}, m_{B^*}, m_{B^*}, q) \\ &\quad + I(m_{B_2}, m_{B^*}, m_{B^*}, q)], \\ \mathcal{A}_{\Upsilon_D \rightarrow \chi_{b1}\omega}^{\text{loop}} &= 2N \sqrt{\frac{10}{3}} g_1 g_4 c_\omega \epsilon_{ijk} \epsilon_{\Upsilon_D}^i \epsilon_\omega^j \epsilon_{\chi_{c1}}^k [I(m_{B_1}, m_B, m_{B^*}, q) + I(m_{B_1}, m_{B^*}, m_B, q)], \\ \mathcal{A}_{\Upsilon_D \rightarrow \chi_{b2}\omega}^{\text{loop}} &= N \frac{2}{\sqrt{15}} g_1 g_4 c_\omega \epsilon_{\chi_{b2}}^{ij} \epsilon_{\Upsilon_D}^i \epsilon_\omega^j [5I(m_{B_1}, m_{B^*}, m_{B^*}, q) - I(m_{B_2}, m_{B^*}, m_{B^*}, q)]. \quad (\text{A.1}) \end{aligned}$$

where $N = \sqrt{MM_\chi}$, with M and M_χ the masses of the initial and final heavy particles, respectively, accounts for the nonrelativistic normalization, q is the magnitude of the three-momentum of the ω

in the rest-frame of the initial particle, and $I(m_1, m_2, m_3, q)$ is the scalar three-point nonrelativistic loop integral, the expression of which can be found in Refs. [8, 15]

$$I(m_1, m_2, m_3, q) = \frac{\mu_{12}\mu_{23}}{16\pi m_1 m_2 m_3} \frac{1}{\sqrt{a}} \left[\arctan \frac{c' - c}{2\sqrt{a(c - i\epsilon)}} + \arctan \frac{2a + c - c'}{2\sqrt{a(c' - a - i\epsilon)}} \right], \quad (\text{A.2})$$

with

$$a = \left(\frac{\mu_{23}}{m_3} q \right)^2, \quad c = 2\mu_{12}b_{12}, \quad c' = 2\mu_{23}b_{23} + \frac{\mu_{23}}{m_3} q^2, \quad (\text{A.3})$$

where $\mu_{ij} = m_i m_j / (m_i + m_j)$, $b_{12} = m_1 + m_2 - M$, $b_{23} = m_2 + m_3 + E_\omega - M$, with M the mass of the initial particle and $E_\omega = (M^2 - M_\chi^2 + m_\omega^2)/(2M)$ the energy of the ω -meson.

The meaning of the velocity v in the power counting can be seen from expanding the loop function around $a = 0$ [17]

$$I(m_1, m_2, m_3, q) = \frac{\mu_{12}\mu_{23}}{16\pi m_1 m_2 m_3} \frac{2}{\sqrt{c} + \sqrt{c'}} + \dots, \quad (\text{A.4})$$

where only the leading order term is kept. Notice that the two square roots inside the arctan functions in Eq. (A.2) correspond to the two cuts in Fig. 1. The one containing $\sqrt{c - i\epsilon}$ is connected to the initial heavy quarkonium and cuts the intermediate states with masses m_1 and m_2 ; the other, connected to the final heavy quarkonium, contains $\sqrt{c' - a - i\epsilon}$ and cuts the intermediate states with masses m_2 and m_3 and the light particle in the final state. It is thus clear that v in the power counting is the average of the two velocities defined through these cuts.

Therefore, although the power counting of this scalar triangle loop is given by $\mathcal{O}(v^{-1})$, the loop function does not diverge even if $M = m_1 + m_2$. Indeed, the triangle loop integral has singularities in addition to the normal thresholds which correspond to the branching points of the cuts. This has been known for a long time [23], and such singularities are called Landau singularities. Landau singularities for a given loop diagram are determined by the solutions of the Landau equations. For the triangle diagram shown in Fig. 1, the leading singularities are determined by the following equation [23]

$$1 + 2 y_{12} y_{23} y_{13} = y_{12}^2 + y_{23}^2 + y_{13}^2, \quad (\text{A.5})$$

where

$$y_{ij} = \frac{m_i^2 + m_j^2 - p_{ij}^2}{2 m_i m_j}. \quad (\text{A.6})$$

In our case, we have $p_{12}^2 = M^2$, $p_{23}^2 = M_\chi^2$ and $p_{13}^2 = m_\omega^2$.

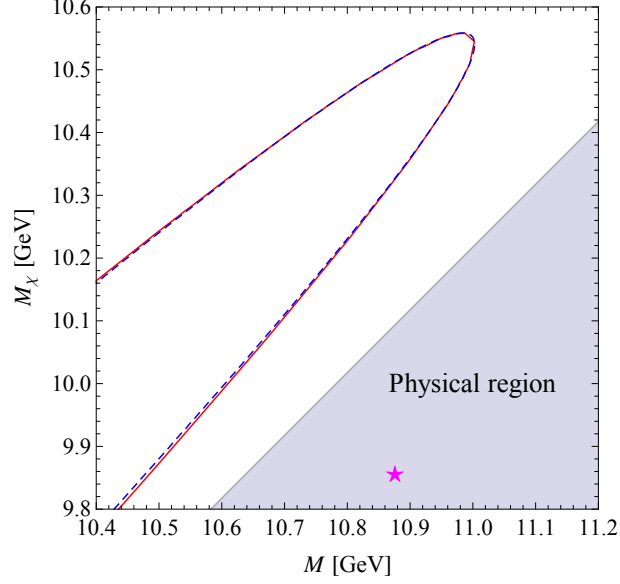


FIG. 5: The Landau singularity of the scalar triangle loop function for the intermediate mesons being $[B_1, \bar{B}, B]$. Here, M and M_χ are the masses of the initial and final heavy quarkonia, and the light particle mass is m_ω . The solid and dashed curves represent the trajectories for the solutions of the Landau equation, Eq. (A.5), and the nonrelativistic equation, Eq. (A.7), respectively. The shaded area given by $M \geq M_\chi + m_\omega$ is the physically allowed region. The star marks the point with $M = M_{\Upsilon(10860)}$ and $M_\chi = M_{\chi_{b0}}$.

As for the nonrelativistic triangle loop function in Eq. (A.2), the triangle singularity occurs when the arguments of the arctan functions take a value of $\pm i$. We find that the singularity equations from both arctan functions are the same, which is

$$(c' - c)^2 + 4ac = 0. \quad (\text{A.7})$$

Notice that this equation is of eighth order in the masses of the initial and final heavy particles. Given a value of the initial mass, one gets eight solutions for the mass of the final heavy particle M_χ . However, since Eq. (A.2) is the expression for the nonrelativistic three-point scalar loop integral, only those solutions of M_χ within the vicinity of $m_2 + m_3$ are valid. The solutions of interest of Eq. (A.7) are very close to those of Eq. (A.5) as can be seen explicitly from Fig. 5. They are not exactly the same because the Landau equations and thus Eq. (A.5) are derived for relativistic propagators, while Eq. (A.7) is obtained from the nonrelativistic loop integral. When the particle masses are real, Eq. (A.7) can only be satisfied when either a or c is non-positive, i.e. $q^2 \leq 0$ or $M \leq m_1 + m_2$. As a result, the Landau singularity is located outside the physical region, as can be seen from Fig. 5. The $\Upsilon(10860) \rightarrow \omega \chi_{b0}$ process, denoted by a star in the plot, is not far from the singularity trajectory. However, for $M = M_{\Upsilon(10860)}$, the loop function does not diverge at the

solutions of Eq. (A.7) because the divergences from both arctan functions cancel with each other in this case.

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